# GHZ Theorems in the Framework of Outcomes in Branching Space-time<sup>†</sup>

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A simple algebraic framework is constructed, in which nonstochastic GHZ-Bell theorems can be analyzed. The framework merges Belnap's *outcomes in branching time* with his *branching space-time* (BST). We show that an important structure in BST, called *the family of outcomes of an event*, is a Boolean algebra. We prove that there is no common cause that accounts for the results of GHZ-Bell experiment but we construct common causes for two other setups.

# **1 INTRODUCTION**

Amazingly enough, philosophers' curiosity about Bell-type theorems seems to derive from what these theorems indeed prove, namely, that some experimentally well corroborated quantum predictions cannot be reproduced by so-called local contextual hidden variable models. Technical as the result is, it may nevertheless be viewed as revealing something about causality, since a premise of the theorems bears close resemblance to what is known as Reichenbach's common cause principle (Reichenbach, 1956). If only it is accepted that, indeed, the common cause principle is operative in the derivations of Bell-type theorems and that it correctly captures phenomena of common causation, the theorems in question can be seen as a refutation of an ideal of causal explanation. Since the theorems are experimentally con-

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firmed, it appears that finally a highly metaphysical issue can be experimentally tested.<sup>3</sup>

Recently, however, the issue of the extent to which causality is involved in the derivations of the Bell-type theorems has become highly controversial,<sup>4</sup> not least because of the intrinsic complexity of the two notions involved: probability and causality. Fortunately, among the Bell-type theorems there is a variety, called after its inventors the GHZ-Bell theorems, that are nonstochastic and thus avoid making use of at least one of the two controversial notions above, yet, like their stochastic cousins, prove that reconstructing quantum predictions by local contextual hidden variable models is impossible (Greenberger *et al.*, 1989).

It is with the eve on these theorems that we set out to develop a framework in which subluminal causality can be analyzed in space-time structures. The proofs of these theorems frequently appeal to modality, as in: "if a different setting were chosen at a distant apparatus, this should not influence the result actually observed by the nearby apparatus." Thus, we are looking for models that will naturally merge modality, space-time, and prohibition of superluminal causal signaling, and, at the same time, will avoid too complicated mathematical machinery. Our inspiration came from Belnap's 'branching spacetime' (BST) (Belnap, 1992) and his 'outcomes in branching time' (Belnap, 1995). It is already known that GHZ-Bell theorems can be analyzed in rather special models of branching space-time (Belnap and Szabó, 1996). Thus, our project is to rigorously define the models of outcomes in branching spacetime and then show that Belnap-Szabó argument can be carried out in this more general framework. Our framework enables us to define the nonstochastic common cause, analyze GHZ-Bell theorems, and prove that there is no common cause that accounts for the results of a GHZ-Bell experiment. On the other hand, we can construct common causes for two other quantum mechanical setups. The question of why some setups allow for common causes whereas others do not can also be tackled. Here, however, as we cannot enter into details, let alone give proofs,<sup>5</sup> we will concentrate on an expository account of our framework.

# 2. THE ALGEBRA OF OUTCOMES IN BRANCHING SPACE-TIME

We will work with structures  $\mathbf{W} = \langle W; \leq \rangle$  such that W is a nonempty set, partially ordered by  $\leq$ . Intuitively, we might think of W as a totality of

<sup>&</sup>lt;sup>3</sup>This view has been advocated by Shimony, see for instance, Shimony (1989).

<sup>&</sup>lt;sup>4</sup>Cf., e.g., Cartwright (1989, Ch. 6), Berkovitz (1995), Butterfield (1992).

<sup>&</sup>lt;sup>5</sup> An extended version of the paper, with all the relevant proofs, is forthcoming (Kowalski and Placek, n.d.).

spatiotemporal points. The relation  $x \le y$  is then interpreted as meaning that x is in the backward light cone of y, and hence might be safely assumed to be a partial order.

Our structures will be required to satisfy certain additional postulates, motivated partly by Belnap and partly by a physical intuition of sorts. Before we state them, it will be convenient to define a couple of auxiliary notions.

Definition 1 (Compatibility). We say that  $x, y \in W$  are compatible iff there is a  $z \in W$  with  $z \ge x$  and  $z \ge y$ .

We write  $x \perp y$  to indicate that x and y are *not* compatible, and we refer to such elements as *orthogonal*.

Next, following Belnap, we introduce some special subsets of *W* called 'histories'. Intuitively, a history is to represent a possible course of events.

Definition 2 (History). A subset h of W is a history iff h is an upwarddirected subset of W, and for all upward-directed  $h' \subseteq W$  we have  $h' \supseteq h$ implies h' = h. In other words, histories are maximal upward-directed subsets of W. We denote the set of all histories by  $\mathcal{H}$ .

Since a course of events may go one way or another, histories split. To exhibit how we think this splitting is effected, let us consider a photon that is approaching a surface of a translucent medium. After hitting the surface, the photon can be either refracted or reflected. Accordingly, we have (a bundle of) histories in which the photon is refracted and (another bundle of) histories in which it it reflected. With some idealization, the histories split at the point of the photon hitting the surface of the medium.

Thus, we define the sets of *splitting points*.

Definition 3 (Splitting points). For any two orthogonal points  $x, y \in W$ , we define  $C(x, y) \subseteq W$  by putting  $z \in C(x, y)$  iff z is a maximal element in  $\{z \in W: z \le x \& z \le y\}$ . If x and y are not orthogonal, we put  $C(x, y) = \emptyset$ .

With splitting points defined, we can state the two conditions we place on the relation  $\leq$ .

- (C1) For any  $x, y, z \in W$ , if  $x \perp y$  and  $z \leq x, z \leq y$ , then there is some  $t \in C(x, y)$  with  $t \geq z$ .
- (C2) For any  $x, y, z, t \in W$ , if  $x \ge z$  and  $y \ge t$ , then  $C(x, y) \supseteq C(z, t)$ .

The first excludes certain "pathological" structures, and brings about the desired effect that any two points a and b belong to a common history, h iff there are no splitting points for them.

The second says that splitting points are retained along histories, that is, if two points belong to different courses of events, i.e., the set of their splitting points is nonempty, and we move further on along one or the other course, then we do not lose any splitting points, although we may gain some new ones.

Definition 4 (Precedence). For  $E, F \subseteq W, x \in W$ , (1) E < x iff  $\forall_{e \in E} e < x$ ; (2) E < F iff  $\forall_{x \in F} E < x$ .

Definition 5 (Relative orthogonality). Elements x, y of W are orthogonal relative to E, written  $x \perp_E y$ , iff E < x, E < y, and  $C(x, y) \cap E \neq \emptyset$ .

Definition 6 (Orthogonal complement). For  $F \subseteq W$ , the orthogonal complement of *F* relative to *E* is the set  $F^{\perp_E}$  such that  $x \in F^{\perp_E}$  iff  $\forall_{v \in F} x \perp_E y$ .

Definition 7 (Outcome). A subset F of W is an outcome of E iff  $F = F^{\perp_{E}\perp_{E}}$ 

The three definitions above lead to a notion that satisfies a number of intuitive requirements for outcomes. The outcome of E is preceded by E and is located as close as possible to E. What the outcomes of E look like crucially depends on whether and, if so, how many, histories split in E.

Definition 8 (Atomic outcomes). e is an atomic outcome of  $E \subset W$  iff (1) e is a nonempty outcome of E and (2) there is no nonempty outcome u of E such that  $u \subset e$ .

Theorem 1. The lattice  $\mathscr{F}_E = \langle F_E, \cup, \cap \bot_E, \Omega_E, \emptyset \rangle$  of outcomes of E is a Boolean algebra, where  $\Omega_E = \{x \in W: E \leq x\}$ . Moreover, the Boolean algebra of outcomes is complete, and whenever it is nontrivial, it is also atomic.

We can also prove a converse result, i.e., that every atomic and complete Boolean algebra can be represented as the family of outcomes of some event in a model of outcomes in branching space-time.

Let us now take a closer look at the outcomes of some specific subsets of W. Since every outcome of E must be preceded by E, for any E containing a pair of orthogonal points, its only outcome is the empty set. The same holds for any subset E of W that is not bounded from above. There are, however, subsets of W that must have nonempty outcomes. We will dub these subsets 'events' and define them as follows:

Definition 9 (Events). We say that  $E \subset W$  is an event if  $E \neq \emptyset$  and  $\exists_{x \in W} E < x$ .

Corollary 1.  $E \subset W$  is an event iff E has a nonempty outcome. Every event  $E \subset W$  has an atomic outcome.

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# 3. COMMON CAUSES IN BST

In this section we will introduce the concept of nonstochastic common cause. We call it 'nonstochastic' because we do not assign probabilities to outcomes, and thus our common causes are designed to account for the nonexistence of certain outcomes rather than some peculiar probability distribution of outcomes. We begin with definitions.

Definition 10 (Consistent subsets of W).  $E_1 \subset W$  is consistent with  $E_2 \subset W$  if there is a history that intersects both  $E_1$  and  $E_2$ ; in symbols,

$$\exists_h (h \cap E_1 \neq \emptyset \& h \cap E_2 \neq \emptyset)$$

We say that  $E_1$  is *inconsistent* with  $E_2$  iff it is not the case that  $E_1$  is consistent with  $E_2$ .

Definition 11 (Spacelike events). The set  $\{E_1, E_2, E_3, \ldots, E_n\}$  of events is spacelike only if

- $\bigcup_{i=1}^{n} E_i$  is an event.
- $E_1$  does not overlap with any outcome of  $E_j$ , i.e., for all outcomes  $o_{E_i}$  of  $E_j$ , we have

$$E_i \cap o_{E_i} = \emptyset$$

The second clause of the above definition is perhaps worth commenting upon. Our BST framework ensures that every point that can be causally influenced by  $E_i$  is in some outcome of  $E_i$ . Accordingly, to say that there is no overlap between  $E_k$  and any outcome of  $E_i$ ,  $k \neq i$ , simply means that  $E_k$  cannot be influenced by  $E_i$ .

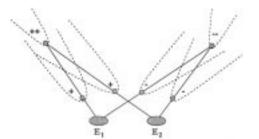
Let us now consider under what circumstances the search for a common cause normally begins. First, there must be an event of the form  $\bigcup_i E_i$ , where  $\{E_1, E_2, \ldots, E_n\}$  is spacelike. As an illustration for n = 2, one may think of the EPR correlations and identify  $E_i$  with the event of measuring the spin projection of a particle at location *i*. Accordingly,  $\bigcup_i E_i$ , i = 1, 2, consists of two measurements performed on a pair of particles at the two spacelikeseparated locations. Since each  $E_i$  and  $\bigcup_i E_i$  are events, they have nonempty sets of outcomes. The set of outcomes of  $E_i$  consists of the outcome containing the observation of + at location *i* and the outcome containing the observation of - at location *i* and the combinations of these two. As for  $\bigcup_i E_i$ , its outcomes are the outcome containing two pluses ++, the outcome containing two minuses --, and the outcomes that are combinations of these two outcomes. Now, contrary to our expectations based on an intuitive combinatorics, an outcome of  $\bigcup_i E_i$  is missing, namely the outcome containing +-. This is because the outcome + of  $E_1$  does not overlap with the outcome - of  $E_2$  (see Fig. 1). Let us emphasize that we use 'missing' in an informal, intuitive sense, not to be confused with 'empty'. The empty set is always an outcome of an event *E*, provided *E* has any outcomes at all, i.e., the set of outcomes of *E* is nonempty. This informal notion of a 'missing' outcome is translated into our set-theoretic framework by noticing that if some outcome of  $\bigcup_i E_i$  is missing, there must be outcomes  $e_1, e_2, \ldots, e_n$  of events  $E_1, E_2, \ldots, E_n$ , respectively, such that  $\bigcap_i e_i = \emptyset$ .

Now, we need to clearly distinguish between a common cause that accounts for missing outcomes of some events constituting a *single* spacelike set and what has recently been called *common* common cause, i.e., an event accounting for missing outcomes of events belonging to many different spacelike sets.<sup>6</sup> The motivation for this distinction should clear by comparing the original EPR argument and the GHZ theorems. In EPR the polarization directions at the two apparatuses are thought of as fixed, whereas in GHZ it is an essential part of the argument that at each apparatus, one of two possible settings can be freely chosen. Consequently, we define below rather the common common cause than the common cause *simpliciter*. To obtain the definition of the latter notion, consider only one spacelike set, i.e., put k = 1.

Definition 12 (Common cause). Let  $F^1 = \{E_1^1, E_2^1, \ldots, E_n^1\}, F^2 = \{E_1^2, E_2^2, \ldots, E_n^2\}, \ldots, F^k = \{E_1^k, E_2^k, \ldots, E_n^k\}$  be spacelike sets in W, and for each  $j = 1, \ldots, k$  let there be outcomes  $e_1^j, e_2^j, \ldots, e_n^j$  of  $E_1^j, E_2^j, \ldots, E_n^j$ , respectively, such that  $\bigcup_i e_i^j = \emptyset$  for all j.

A common cause *CC* that accounts for these k facts, that  $\bigcup_i e_i^j = \emptyset$  ( $j = 1, \ldots, k$ ), is an event that satisfies:

- $\forall_{i,j} CC < e_i^j$  [causal precedence].
- Every history that contains CC contains  $\bigcup_i E_i^j$  for some j [conservation].



**Fig. 1.** Each event  $E_i$ , i = 1, 2, has two atomic outcomes that contain + and -, respectively. Atomic outcomes of events  $E_1 \cup E_2$  start with either -- or ++. There is no outcome of  $E_1 \cup E_2$  containing -+ (or +-). The straight lines represent the relation of precedence, <.

<sup>6</sup>We have taken the term 'common common cause' from Hofer Szabó et al. (1999).

For each *j* ≤ *k*, every atomic outcome of *CC* is inconsistent with at least one *e<sup>i</sup><sub>i</sub>*, *i* ≤ *n* [screening off].

The first clause above is a natural requirement that a common cause precedes its putative effects. The second bars such "Pickwickian" common causes that would permit one to bypass all the events whose outcomes the purported common cause is supposed to bring about. Finally, the third clause says more about what a common cause should look like and gives a minimal answer to why  $\bigcap_i e_i^j = \emptyset$  for all *j*.

Conservation has a significant implication that is the content of the following lemma.

*Lemma 1.* Every outcome  $o_{CC}$  of *CC* is consistent with at least one outcome  $o_{\cup_i E_i^j}$  of  $\bigcup_i E_i^j$  for some  $j \le k$ .

Our observation is that there is always a common cause for missing outcomes of events forming a single spacelike set. Since the formal argument is rather tortuous, here is a simple paper-and-pencil construction. Suppose  $\{E_1, E_2, \ldots, E_n\}$  is a spacelike set, so that  $\bigcup_i E_i$  is an event and we search for a common cause of missing set  $e^* = \bigcap_i e_i$ , where  $e_i$  is an outcome of  $E_i$ for i = 1, ..., n. First, to satisfy causal precedence, we locate CC so that any two atomic outcomes  $e_i$  and  $e_i$  and CC form an upward fork (i.e.,  $\neg \exists_{x \in w}$ )  $CC < x \& x < e_i \& x < e_j$  for any  $i \neq j$ ). This instruction takes care of causal precedence. Second, we make sure that there is no point in W that is not in an outcome of some  $E_i$  and is preceded by CC. This, together with the former instruction, satisfies conservation. It remains to be shown that CC so constructed satisfies screening off. Observe that in order for an atomic outcome of CC to be consistent with every  $e_i$ , where  $\bigcap_i e_i = \emptyset$ , histories that contain  $e_j$ , and histories that contain  $e_j$ ,  $i \neq j$ , cannot split in CC. However, the way we located CC ensures that these two bundles of histories do split in CC. The argument can be easily extended so as to apply to cases with any larger number of missing sets of the form  $\bigcap_i e_i$ .

Once there are several spacelike sets  $F^j = \{E_1^j, E_2^j, \ldots, E_n^j\}, 2 \le j \le I$ , each with at least one missing outcome  $\bigcap_i e_i^j = \emptyset$ , satisfaction of the conditions of the common cause ceases to be a trivial matter, as the GHZ theorems testify.

# 4. IS THERE A (COMMON) COMMON CAUSE IN THE BELL-GHZ SETUP WITH THREE PARTICLES?

Consider now Mermin's *Gedanken* experiment (Mermin, 1990). Each particle from a given trio flies away from a source toward one of three stations, the angular distance between any two stations being 120°. At each

station one of two parameters x and y, call them directions, is selected and the passage of a particle through a station brings about one of two results, say + or -. Finally, a triple of such pluses and minuses is registered. To introduce the notation, here is an example:

- $x_i$  and  $y_i$ , where i = 1, 2, 3, are events of setting, respectively, direction x and y at station *i*.
- x<sub>1</sub> ∪ x<sub>2</sub> ∪ y<sub>3</sub> is the event consisting of the three events of setting the corresponding directions at the three stations. We will use the abbreviation E<sub>x,x,y</sub> <sup>df</sup> = x<sub>1</sub> ∪ x<sub>2</sub> ∪ y<sub>3</sub> and analogously for other events of this kind.
- $x_i^{\pm}(y_i^{\pm})$  is an outcome of event  $x_i(y_i)$ ; it starts with result  $\pm$  registered at station *i*, with the direction set at x(y) at the station.
- $\langle x_1^+, x_2^-, y_3^+ \rangle$  is an 'observation event' that consists of registering the triple of pluses and minuses: at station 1 result + with the direction set to *x* and at station 2 result with the direction set to *x*, and at station 3 result + with the direction set to *y*. There is a straightforward link between the existence of the observation event and the intersection of outcomes of the corresponding directions setting being non-empty. For instance, there exists observation event  $\langle x_1^+, x_2^-, y_3^+ \rangle$  if and only if  $x_1^+ \cap x_2^- \cap y_3^+ \neq \emptyset$ .

Now, we have the following stipulations that are either motivated by actual tests of Bell-type theorems or derive from quantum theory:

Setup Stipulation 1. Every set  $\{x_1, x_2, x_3\}$ ,  $\{x_1, x_2, y_3\}$ ,  $\{x_1, y_2, x_3\}$ ,  $\{y_1, x_2, x_3\}$ ,  $\{y_1, y_2, x_3\}$ ,  $\{y_1, x_2, y_3\}$ ,  $\{x_1, y_2, y_3\}$ , and  $\{y_1, y_2, y_3\}$  is spacelike. This assumption is motivated by Aspect *et al.*'s test of a Bell-type theorem, in which the directions at the measuring devices were set only when the particles were sufficiently distant from one another and close enough to the measuring devices (Aspect *et al.*, 1981, 1982).

Setup Stipulation 2. The observation event is contained in a history if and only if this history intersects each corresponding outcome of the events of direction setting. In symbols, for the event  $\langle x_1^-, x_2^+, x_3^- \rangle$ 

 $\langle x_1^-, x_2^+, x_3^- \rangle \subset h$  iff  $h \cap x_1^- \neq \emptyset \& h \cap x_2^+ \neq \emptyset \& h \cap x_3^- \neq \emptyset$ 

Setup Stipulation 3. If a history contains  $x_i(y_i)$ , it must go through either  $x_i^+(y_i^+)$  or  $x_i^-(y_i^-)$ .

*QM Stipulation.* QM imposes a constraint on what triples of pluses and minuses can be observed<sup>7</sup>:

<sup>7</sup>To see how this constraint arises, consult, for instance, Mermin (1990).

- x<sub>1</sub><sup>k</sup> ∩ x<sub>2</sub><sup>l</sup> ∩ x<sub>3</sub><sup>m</sup> ≠ Ø iff k × l × m = −1.
  y<sub>1</sub><sup>k</sup> ∩ y<sub>2</sub><sup>l</sup> ∩ y<sub>3</sub><sup>m</sup> ≠ Ø iff k × l × m = −1.
- Intersection of outcomes with x and y mixed is nonempty iff  $k \times k$  $l \times m = 1.$

Here k, l,  $m = \pm 1$  (more precisely: k, l,  $m \in \{+, -\}$  and we assume that pluses and minuses behave according to the usual classroom arithmetic, that is, two minuses yield a plus).

Idealization. The assumptions above guarantee that every outcome of  $E_{*,*,*}$  contains at least one observation event of the appropriate kind and permitted by quantum theory. To spell it out for  $E_{x,x,x}$ , every outcome of  $E_{x,x,x}$ must contain at least one event  $\langle x_1^i, x_2^j, x_3^k,$  where  $i \times j \times k = -1$ . This still leaves open the possibility that although a history contains event  $E_{***}$ , it does not go through any observation event of the corresponding kind. To exclude this, we assume, as an idealization, that every outcome of  $E_{*,*,*}$  starts with at least one observation event of the corresponding kind. This entails that every history that goes through an outcome of  $E_{*,*,*}$  contains an observation event of the appropriate kind and permitted by QM Stipulation.

No Conspiracy. The outcome of a common cause that should account for missing observation events can be thought of as a set of instructions carried by a triple of flying particles. The instruction for a given particle says whether + or - would be displayed at the station, given that direction x or y is chosen there. Now, to exclude conspiracy, we assume that the instruction carried by the incoming particle cannot be changed by a swap of direction settings at the apparatus. In the framework of BST, this translates into:

Every outcome of common cause CC should be consistent with every  $x_i$  and  $y_i$ , i = 1, 2, 3.

We have come to the central theorem of this section.

Theorem 2. There is no single event CC that is a common cause for every missing outcome of each of  $E_{x,x,x}$ ,  $E_{x,x,y}$ ,  $E_{x,y,x}$ , and  $E_{x,y,y}$ .

*Proof.* Lemma 1 implies that every atomic outcome  $o_{CC}$  of CC is consistent with an outcome of  $E_{x,x,x}$  or an outcome of  $E_{x,x,y}$  or an outcome of  $E_{x,y,x}$ , or an outcome of  $E_{x,y,y}$ . Since (by causal precedence)  $CC < x_l^{\pm}$  for l = 1, ..., 3 and (by QM Stipulation) for some *i*, *j*, *k* such that  $x_1^i \cap x_2^j \cap x_3^k \neq \dots$  $\emptyset$ , there is an atomic outcome of CC that is consistent with  $E_{x,x,x}$ . By Idealization and QM Stipulation, there must be history h that intersects  $o_{CC}$  and contains event  $\langle x_1^i, x_2^j, x_3^k \rangle$ , where  $i \times j \times k = -1$ . By Setup Stipulation 2, *h* must go through each outcome  $x_1^i$ ,  $x_2^i$ , and  $x_3^k$  and hence:

•  $o_{CC}$  is consistent with each  $x_1^i$ ,  $x_2^j$ , and  $x_3^k$ .

Consider next  $E_{x,x,y}$  and take the set  $x_1^i \cap x_2^j \cap y_3^k$ , which is missing by QM Stipulation. By screening off and the fact above:

•  $o_{CC}$  is inconsistent with  $y_3^k$ .

However, by the requirement of no conspiracy,  $o_{CC}$  must be consistent with  $y_3$  and hence, by Setup Stipulation 3,

•  $o_{CC}$  is consistent with  $y_3^{-k}$ 

In the next step, consider event  $E_{x,y,y}$  and set  $x_1^i \cap y_2^{-j} \cap y_3^{-k}$ , which is missing (by QM Stipulations). By screening off and the (in)consistencies established above, we have:

•  $o_{CC}$  is inconsistent with  $y_2^{-j}$ .

Finally, let us focus attention on  $E_{x,y,x}$  and  $x_1^i \cap y_2^j \cap x_3^k = \emptyset$ . Since  $o_{CC}$  is consistent with  $x_1^i$  and  $x_3^k$ , by screening off it must be that:

•  $o_{CC}$  is inconsistent with  $y_2^j$ .

However, inconsistency of  $o_{CC}$  with both  $y_2^{-j}$  and  $y_2^{-j}$  implies (by Setup Stipulations 2 and 3) inconsistency of  $o_{CC}$  with  $y_2$ , and this contradicts the requirement of no conspiracy.

Note the power of the requirement of no conspiracy. Indeed, without this condition it is easy to construct a common cause for the setup considered. It suffices to postulate an event that precedes every  $x_i^{\pm}$  and  $y_i^{\pm}$ , i = 1, 2, 3, for which also conservation holds.

# 5. SOME OTHER RESULTS

In the framework presented above we define certain setups that can arguably be taken for models of other quantum experiments. For instance, we can prove:

Theorem 3:

- 1. There is no (common) common cause in the GHZ-Bell setup with four particles.
- 2. There is a common cause for the EPR (perfect) correlations.
- 3. There is a common cause for the GHZ-Bell argument with three particles if directions are fixed.

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